

# Exhibit C

## Response to Reports of Alford and Trende

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I am a Professor of Mathematics and a Senior Fellow in the Jonathan M. Tisch College of Civic Life at Tufts University. I have previously filed several reports in this case, and my full credentials and CV are available in prior filings. The current report responds to the expert report of John Alford dated July 18 and the expert report of Sean Trende dated July 23. I will refer below to my own earlier reports dated May 23 and June 20.

### Part I

## Response to John Alford

### 1 Summary

In his report, Dr. Alford relies heavily on my earlier work in this case to study recent Democratic primary elections in Texas.

I will quote Dr. Alford at length:

Duchin's **coverage is broad** [in] terms of Democratic primaries, with elections covering the entire decade from 2012-2020, and full ethnic coverage (Black, Hispanic, and Asian). This is critical, as one of the clear functions of the Democratic primary analysis is to assess the degree to which Blacks, Hispanics, and often Asians are sufficiently cohesive that they can be treated as a single coalition for Gingles 1 purposes. In addition, Duchin offers what is **arguably the most sophisticated application of current EI methodology**. As such, the discussion of Democratic primaries that follows will rely on Duchin's analysis and will be presented using her results. As noted above, the differences across experts in mode of EI analysis does not appear to have any systematic impact on the substantive results, and so to make it as clear as possible that **my only dispute with Duchin concerns interpretation**, I will rely on her analysis for my comments and conclusions regarding Democratic primaries. (Alford p13-14, emph. added)

In other words, Dr. Alford stipulates to the thoroughness and soundness of my methods and differs only in how to interpret the results. Despite this vote of confidence, he has completely neglected the element of my analysis that I insisted was central: the signals of certainty and uncertainty. Furthermore, he has imposed a definitional cutoff of group cohesion (60% of the group's vote going to a particular candidate) that is both arbitrary and unreasonable, especially in primary elections.

### Major issues with analysis in Alford report

- **Uncertainty.** Dr. Alford treats point estimates—single numbers that represent averages over a distribution of possibilities—as true, stand-alone reflections of group preference, ignoring the confidence estimates. In many primaries, the uncertainty indicators tell us that the noise is overwhelming the signal of group preference.
- **Candidates of choice.** The Alford report sets a 60% cutoff for group cohesion, even in primaries. This is patently unreasonable in primaries, which can frequently have six or more candidates, and it is misleading for him to claim support for this standard by citing other experts' discussions of general elections. It is far more reasonable to actually make use of the confidence estimates to illuminate which elections have clear leaders in group preferences.
- **White bloc voting.** The Alford report argues that Gingles 3 is not satisfied because White voters in Democratic primaries often vote together with POC voters (people of color, i.e., members of racial and ethnic minorities). But this misunderstands the way in which my evidence supports the TX-NAACP legal argument: Black, Hispanic, and Asian voters show signs of frequent cohesion among primaries where a reasonably clear signal is present, and are undeniably cohesive in general elections. Meanwhile, White voters consistently block the preferences of minority voters in general elections. That is, only a small share of White voters is present in Democratic primaries, and their preferences in those primaries are not relevant to the argument.<sup>1</sup>

## 2 The role of uncertainty in inference techniques

"Statistical inference" is a broad name for methods that estimate quantities that were not measured directly, using patterns in related data. Sometimes we have so little of the right kind of data that the uncertainty overwhelms the estimate.

For example, imagine that someone has asked you to guess a number between 1 and 100. If you know a great deal about the process used to generate the number, you might build a model that gives you high confidence that you can estimate the number. But if you have no further information at all, then it could be any of the 100 possible values. Absent any other data or hypotheses, it might be reasonable to build a model in which all values are equally likely.

Dr. Alford's method of reporting point estimates and ignoring uncertainty would be analogous to simply averaging all the numbers from 1 to 100 and reporting the output as a statistical estimate of  $\frac{1+2+\dots+100}{100} = 50.5$  for the unknown number. A far better answer in this case would be that there is not enough information to give a meaningful estimate. Figure 1 below, reproduced from my June 20 report, illustrates visually that it would be highly misleading to pay attention only to averages and not to the corresponding confidence and uncertainty. Reporting averages alone can hide big differences in confidence.

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<sup>1</sup>This was spelled out in my earlier supplemental report: "I note that though the Other (mostly White) Democrats generally show high levels of agreement with the coalition minority groups in primaries in Denton-Wise, Lubbock, and Brazoria, this is unrelated to a Gingles 3 analysis. In particular, Black-Hispanic-Asian coalition voters are cohesive in primaries and highly cohesive in general elections, but their preferences are consistently blocked by the White majority in general elections." (June 20, p5)

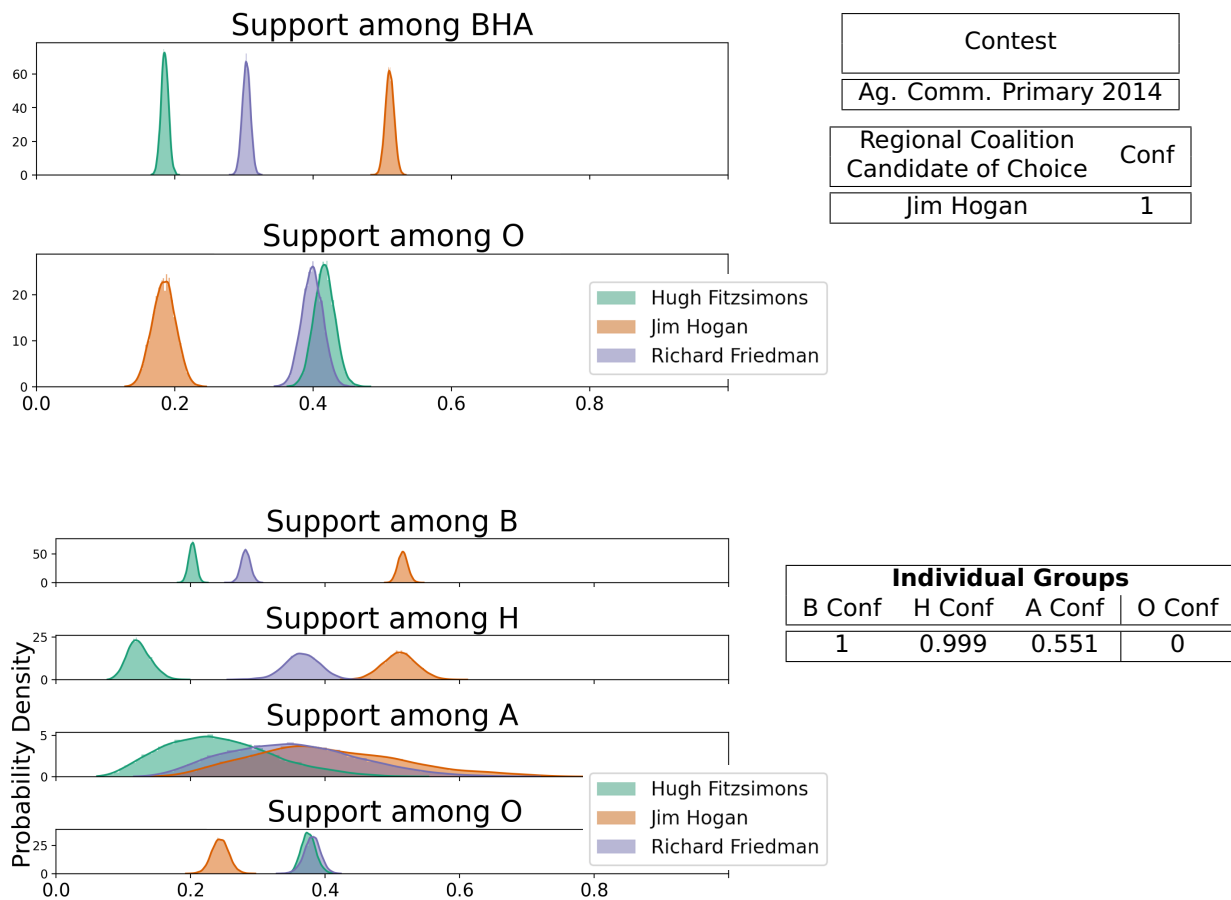


Figure 1: This example, repeated from Figure 1/Table 3 of my June 20 report, shows the findings of an EI run on the Agriculture Commissioner Democratic Primary election from 2014 in Harris/Fort Bend. Dr. Alford would label this an election where no group has a clear candidate of choice, while I label high confidence that Hogan is the choice for Black and Hispanic voters as individual groups and for coalition (Black+Hispanic+Asian) voters together. Furthermore, Dr. Alford considers only averages, ignoring low confidence; in this case the width of the Asian support curve around its average of 40.45% reflects that it could well be under 25% or over 60%. See also Figure 4 for another example.

EI works to infer the turnout and the candidate preferences of racial and ethnic groups by looking at the cast vote patterns across precincts. As I explained in my initial report:

EI is at its strongest when the number of racial groups and candidates is modest, the number of precincts is large, and there is considerable variation in demographics... Primary elections give a far weaker signal of voting patterns by race because of low turnout and many candidates; runoff elections have only two candidates but have the lowest turnout of all. (May 23, p26)

Indeed, I noted that general elections in Texas have often reached turnout levels of 50% of CVAP, which gives a rich dataset for analysis. By contrast, recent primaries can fall as low as 200,000 votes cast statewide—typically hovering around 5% of CVAP and sometimes dipping well lower. Since the information is so limited, patterns by precinct

would have to be extremely sharp to produce any reliable inferences.

As predicted, we do see significant uncertainty in the EI outputs of primary and runoff elections. However, there are also useful signals to be found there. The outputs reported at length—highlighting the confidence estimates—in my June 20 report.<sup>2</sup>

### 3 Identifying candidates of choice

#### 3.1 General election patterns are clear

As Dr. Alford concedes, "As a glance down the rows in Table 1 clearly illustrates that general elections show very clear and remarkably stable evidence of partisan voting" (Alford p5). Of course, by "partisan voting," he means that the minority groups show consistent preference for one party while the majority group shows an equally consistent preference for the other. Later, he calls this partisan split "dramatic" (Alford p7). The relevance of this pattern to a Gingles 2-3 inquiry is a matter for legal determination. The data can only take us so far: for these areas of Texas, everyone agrees that general elections are polarized along racial lines, and that the split tracks with party preference.

#### 3.2 Race of the candidates

However, Dr. Alford's report seems to advance a theory that I have not seen before in the scholarly literature. Namely, he seems to argue that true racial polarization would have to implicate the *race of the candidate*; that is, candidates of choice should belong to the racial group under consideration. His Tables 1, 2, 3, and 4 are all designed to highlight whether the race of the candidate has an appreciable impact on their support by various groups. And he writes: "If racial attitudes were so powerful in driving party choice among voters that they override other political considerations like ideology or policy preferences, why is it that these same voters, once in the appropriate party, no longer betray any aggregate positive or negative response to the race of candidates in general elections?" (Alford p13)

In my understanding, the Gingles 2 precondition is met by showing cohesion among the relevant minority groups as to a candidate of choice, irrespective of the candidate's race or ethnicity. While candidate race can play a role in determining the relative value

<sup>2</sup>Here is a brisk top-line summary. **Tarrant/Dallas:** Black voters' preference is clear (at the level I called "very likely," namely at least 90% of draws agree) in 16 out of 18 primary and runoff elections; Hispanic voters' preference is clear in 16 out of 18; Asian voters' preference is clear in 7 out of 18 elections. Black and Hispanic preferences clearly agree in 11 elections out of 18, likely agree in 2, likely disagree in 2, and are clearly divergent in only 3 elections out of 18. (Three contests did not complete with clean diagnostics.) **Harris/Fort Bend:** Black voters' preference is clear in 20/20 elections; Hispanic voters' preference is clear in 18/20 elections; Asian voters' preference is clear in 12/20 elections. Black and Hispanic preferences clearly agree in 10 elections out of 20, likely agree in 2, and are clearly divergent in 8 elections out of 20. (One contest did not complete with clean diagnostics.) **Denton/Wise:** Black voters' preference is clear in 11/21 elections; Hispanic voters' preference is clear in 10/21 elections; Asian voters' preference is clear in 11/21 elections. Black and Hispanic preferences clearly agree in 4 out of 21, likely agree in 4, likely disagree in 6, and clearly disagree in only 1. In the eight elections with the clearest EI outputs, all three groups likely agree in 5 out of 8, and two of the three groups likely agree in the remaining 3. **Lubbock:** Black voters' preference is clear in 4/21 elections; Hispanic voters' preference is clear in 6/21 elections; Asian voters' preference is clear in 1/21 elections. Black and Hispanic preferences clearly agree in 2 out of 21, likely agree in 8, likely disagree in 4, and clearly disagree in 0. In the eight elections with the clearest EI outputs, all three groups likely agree in 4 out of 8, and two of the three groups likely agree in the remaining 4. **Brazoria:** Black voters' preference is clear in 8/21 elections; Hispanic voters' preference is clear in 6/21 elections; Asian voters' preference is clear in 14/21 elections. Black and Hispanic preferences clearly agree in 3 out of 21, likely agree in 7, likely disagree in 6, and clearly disagree in only 1. In the eight elections with the clearest EI outputs, all three groups likely agree in all 8.

of various elections as evidence, I do not believe that cohesion requires a preference for members of one's own race. On the contrary, the logic of a coalition claim, while it does not foreclose some level of within-group preference, is strengthened when the individual minority groups also show a tendency to coalesce.

### 3.3 Determining a candidate of choice

The Alford report advances a threshold for identifying candidates of choice that is not standard even for general and runoff elections and is fully inappropriate for primaries: the point estimate of a group's support for the candidate should be over 60%, regardless of other circumstances. All of page 4 of the Alford report is dedicated to defining and defending this "60% cohesion threshold."

Justification includes this passage: "If the 'Latino candidate of choice' is defined as the candidate that gets the majority of Latino vote, then in a two-party contest there is always a preferred candidate (absent an extremely rare perfect tie) and that candidate always has a majority of Latino votes by definition, and thus all two-party contest [sic] exhibit minority cohesion. Other experts have also generalized the problem to multi-party contests by defining both the candidate of choice and the baseline for cohesion as a simple plurality, again a test that is always met. It should be obvious that this is incompatible with the Gingles test because a threshold test that is always met is not a threshold test, and therefore Gingles 2 would be superfluous" (Alford p3-4). Once again, this is a worldview that admits no uncertainty. Under realistic conditions of estimating electoral support, we frequently find that a group's preference can not be reasonably inferred. This could be due to a roughly even split of support (as for White/Other voters in Figure 1, who are estimated to choose Fitzsimons and Friedman in about equal shares) or could be due to high uncertainty (as for Asian voters in the same figure).<sup>3</sup>

A much more reasonable standard for the group to have a candidate of choice—that works across election types and other variables—is to draw repeatedly from the probability distribution learned by EI and to see that a particular candidate is the leader all, or handily most, of the time. Returning to Figure 1, this standard allows us to conclude with a high degree of confidence that some groups of voters—Black voters taken alone, Hispanic voters taken alone, and Black+Hispanic+Asian voters taken together—prefer Jim Hogan over the other candidates.

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<sup>3</sup>A second example of his reasoning is found here: "If we put cohesion on a proper 100% scale from the complete absence of cohesion at 50% to fully cohesive at 100%, 75% cohesion is at the midway point in terms of relative cohesion" (Alford p4). Besides the usual flaw of assuming the point estimate tells the whole story, 50% support is not "the complete absence of cohesion." On the contrary, 50% in a primary is fairly high support: Texas holds runoff elections in the first place because it is often the case that no candidate receives 50% of primary votes.

## 4 Conclusion

Compounding the secret ballot with extremely low turnout, primary and runoff elections present challenges to all methods of statistical inference. A careful treatment of estimation and uncertainty in EI outputs shows that we can identify at least eight primary and runoff elections in each county cluster that give reasonably strong signals of preference for Black and Hispanic voters, and sometimes for Asian voters as well. Where the statistical signal allows a conclusion, there is significant agreement between the three groups. This is not universal, and there are certainly some contests in which the groups' preferences diverge. But on the whole, and taken together with resounding agreement in the general elections, I find this to be compelling evidence in favor of coalitional voting behavior.

## Part II

# Response to Sean Trende

## 1 Summary

The Trende report of July 23 is mostly organized by level of redistricting (§V: Congress, §VI: state Senate, §VII: state House) and within those sections, mainly by region and by district. The main opinions are summarized in Mr. Trende's two top-line bullet points (§III, p7). Paraphrasing, these argue that (1) the state's enacted map should be understood as a partisan gerrymander, and that (2) the Gingles 1 demonstration maps proposed by plaintiffs' experts are not sufficiently compact, even though they are more compact than the enacted plans.

Taken as a whole, the Trende report supplies ample evidence to corroborate my own findings of **dilution of minority electoral opportunity** as well as **failures of compactness** in the state's enacted maps. However, the report argues that neither the dilution nor the non-compactness is meaningful in the context of this litigation, because both are explained by an overriding set of partisan objectives on the part of the state. In the case of minority opportunity, this fits into a discussion of race vs. party in recent legal history. In the case of compactness, I believe this to be a novel argument.

### 1.1 Party, not race

Throughout the report, Mr. Trende often effectively concedes that the state's plan dilutes minority opportunity to elect, in that it provides markedly less opportunity than a significant share of comparator maps drawn with no racial or partisan data. However, his prevailing argument seems to be that the minority vote dilution is incidental to partisan aims, and thus it is permissible.

For example, in section V.A, Mr. Trende suggests that the contention that districts were drawn with racial intent is undermined if they were "drawn to substantially improve the political advantage of the Republican Party" (Trende p9).

He then presents a large collection of red/blue maps (Figs. 2, 4, 5, and many more) that show district lines on a background in which precincts are colored by the Biden share of the major party vote. He offers these up for "visual inspection" (his phrase, p34) to suggest that the lines are drawn with strong partisan motivation. The resulting visuals are far from striking, especially when compared to racial dot densities. (See Figure 2.)

It is difficult to ascertain whether Mr. Trende's discussion of partisanship over race is directed at the intent claims, the racial gerrymandering claims, or the Section 2 vote dilution claims in this case. But, however they are directed, it is my opinion that the evidence clearly shows that race was used by the state as a sorting mechanism. Below, I will support this with additional evidence that the extreme outlier status in racial statistics is not explained by partisan performance (§3.2). Furthermore, nothing in the Trende report disputes or undermines the clear and strong evidence that the **results** of the new lines are dilutive, compared to a neutral baseline built under accepted districting principles. On the contrary, Mr. Trende's materials are entirely consonant with that finding.



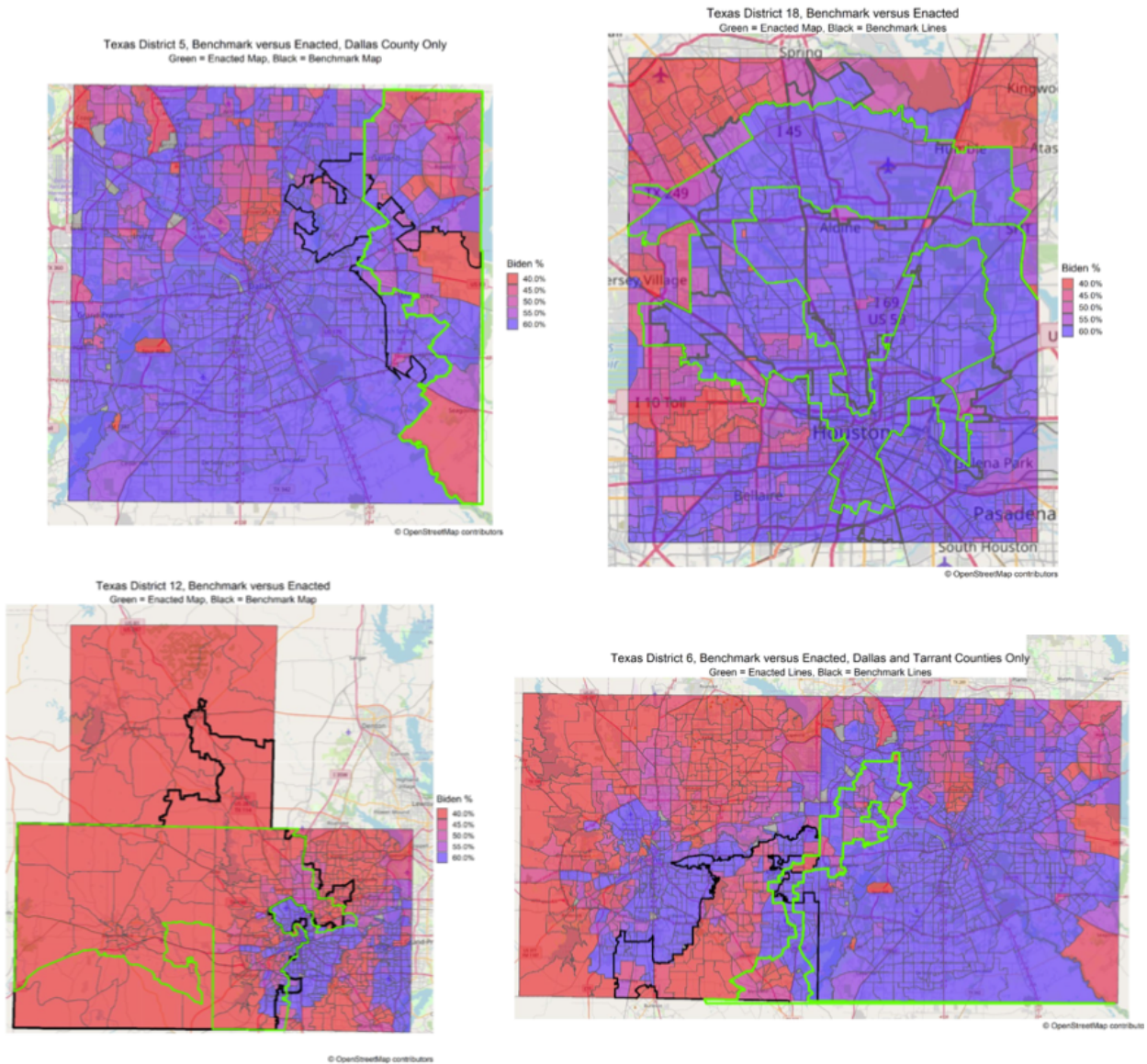


Figure 2: These figures from the Trende report show Congressional boundaries against Biden vs. Trump outcomes by precinct. I believe they are meant to illustrate that partisan concerns dictated the movement of the boundary lines. If the movement from black lines to green lines followed a clear pattern of red and blue, the visual evidence would support that finding; that is not what we see in these plots, where the changes appear to have no strong relationship to the red and blue shading. The Trende report includes over 60 images in this style, typically offering equally weak support for the principle of partisan sorting. This visual also illustrates the limited usefulness of analyzing the partisanship of only the blue precincts, rather than the net partisanship of the districts. (Discussed below in §3.1)

## 1.2 Compactness

In my May 23 report, I offer Gingles 1 demonstrative districts for Congress in clusters that I call C1 (Tarrant/Dallas) and C2 (Harris/Fort Bend); for state Senate in S1 (Tarrant/Dallas) and S2 (Fort Bend); and for state House in H1 (Tarrant), H3 (Denton/Wise), H4 (Lubbock), and H5 (Brazoria). In my supplemental report of June 20, I offer additional demonstration plans in C2, S2, and H4, to emphasize the potential for other priorities such as increased *electoral effectiveness* for minority groups to be emphasized in remedial maps, above and beyond crossing demographic thresholds. This makes eleven modular, regional demonstration plans in all. What makes them modular is that each one replaces a cluster of districts from the enacted plan with an alternative that covers *exactly the same terrain*. This means that they are on the whole quite deferential to choices made by the state, and can be adopted without causing changes to ripple across the whole state of Texas.

The Trende report criticizes my alternative plans for being insufficiently compact, despite the fact that ten out of eleven are *more compact* overall than the state plan in the same terrain.<sup>4</sup>

Mr. Trende may not have realized that my plans are exact replacements for collections of districts drawn by the state. So, for instance, when he colorfully writes that my CD 32 in C1-ALT "takes on a shape resembling Carmen Miranda putting a cake in the oven," he seems to be unaware that more than half of the district's perimeter *follows lines from the state's plan*. See Figure 3 below, which clearly demonstrates that this CD 32 is part of a limited re-drawing that reduces the number of county traversals and substantially improves the bizarre and non-compact shapes of districts in the grouping overall.

In Mr. Trende's words: "A state's enacted plan, however, is not necessarily a good comparator [for Gingles 1 purposes]. If a state pursues a partisan gerrymander, (and its partisan goals outweigh compactness goals), then its enacted districts may not be very compact" (115). In other words, he is suggesting that the Gingles 1 demonstration districts are to be held to a higher standard of compactness than the districts legally enacted for use by the state, under which actual elections will be conducted. But in my understanding, adherence to traditional districting principles, including compactness, is an important factor that the Court will consider in assessing unconstitutional racial gerrymandering. For this reason, it can be highly useful to identify whether intertwined racial and partisan goals indeed outweighed traditional principles like compactness, as he allows here.

Finally, the Trende report conflates two kinds of compactness repeatedly: the shapes of districts versus what is sometimes called "functional compactness" or "cultural compactness"—whether the districts unite reasonably similar and geographically proximal communities. In terms of functional compactness, my alternative districts stretch no further than the ones they replace in the state's plan, because of the significant deference reflected in the modular design. The communities connected in the districts are typically within a single county, assuaging any reasonable concerns about gross regional differences, transportation, media markets, and so on. This also means that no additional counties are split to form the alternative districts; indeed, in many cases I have reduced the number of county traversals. In other words, the alternative districts do not display any signs of functional non-compactness beyond those that follow from my deference to the state's plan.

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<sup>4</sup>The main compactness metric that I relied on is the most common one used in redistricting litigation: the average Polsby-Popper score. The improvement of my alternatives over the enacted plan is significant enough in most cases that it would likely hold up in any other reasonable score.

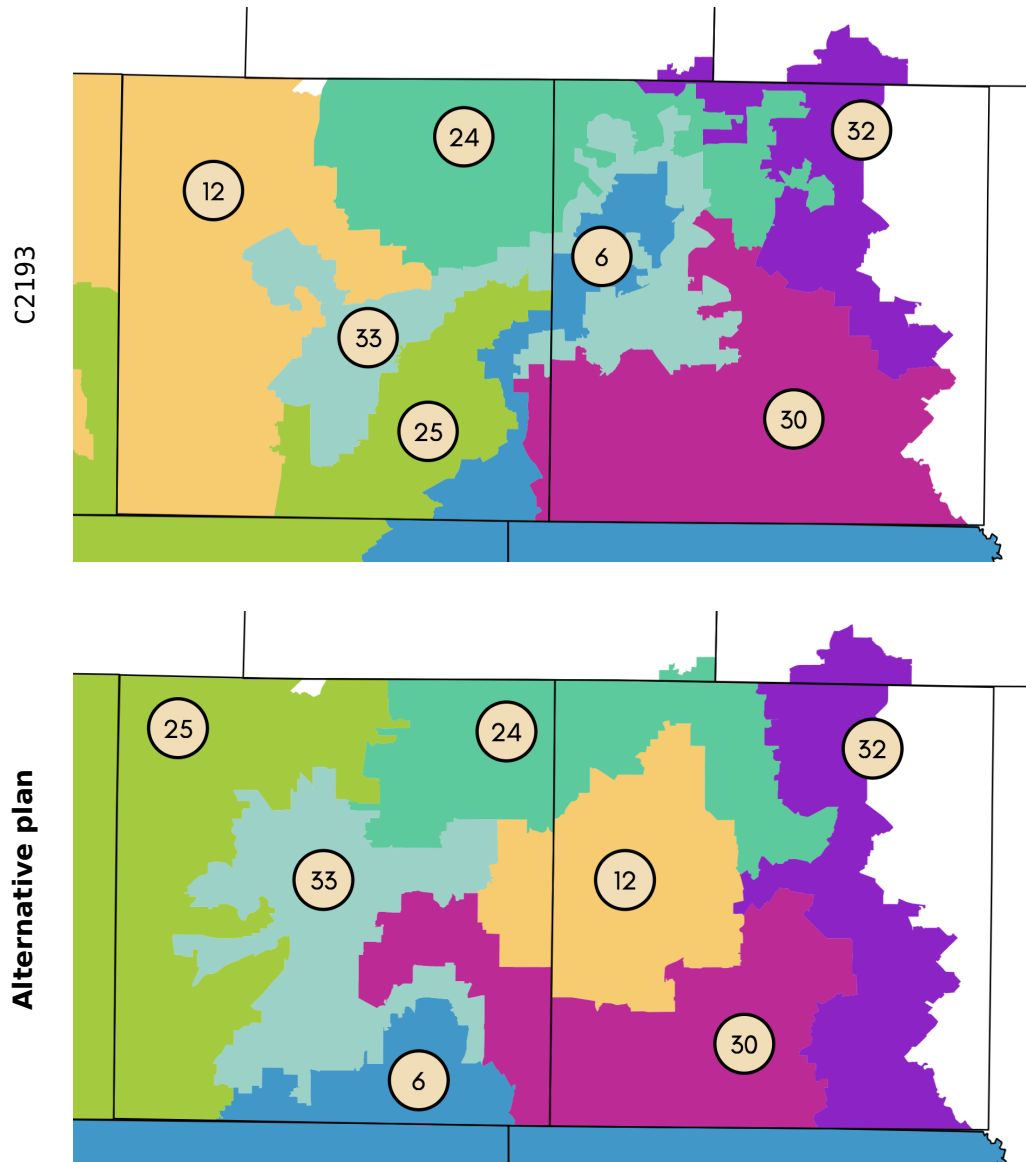


Figure 3: A close-up view of how Congressional districts meet Tarrant and Dallas Counties in the state's plan (top) and the alternative plan (bottom). Reproduced from May 23 report, Figure 3. The alternative plan clearly presents a major improvement on district compactness, while keeping the outer boundary of the cluster of districts the same as it was in the enacted plan C2193.

## 2 Points of agreement

### 2.1 The state's enacted plans are demographic outliers

The format of the Trende report makes it very difficult to tell exactly which districts feature in his "dotplots" and other tables and figures.<sup>5</sup> For instance, his Table 1 and Table 3 list eleven DFW-area Congressional districts (Trende pp35,37), his Table 2 lists nine (p36), his Figure 23 shows twelve (p38), and his subsequent Figure 26 shows only seven, but does not indicate which ones (p42). My Harris/Fort Bend cluster has eight Congressional districts, so none of these offers an apples-to-apples comparison.

Nonetheless, Mr. Trende declares his results to be "substantially similar" to my ensemble findings. He is using an implementation of an algorithm called SMC that has not yet completed peer review for its mathematical correctness, and I am offering no opinion about the soundness of either the algorithm or the implementation.<sup>6</sup> His conclusions are based on his (strictly qualitative) discernment that the "political features" are "at least as extreme" as the outlying racial attributes.<sup>7</sup>

On several occasions, Mr. Trende's own ensembles show that the state's enacted map creates fewer majority-POC districts than even the median of a neutral ensemble. For instance, Figure 49, which is once again not clearly captioned but seems to present the results of scrambling eight Congressional districts in the Houston area, suggests that *two* districts—those indexed #3 and #4—are majority-minority in more than half of his comparator plans, but are majority-White in the enacted plan.

Furthermore, the report is inconsistent in when it even offers a juxtaposition of racial and partisan dotplots. By the time we get to state House districts, such as Denton/Wise (Trende Fig. 133, p183) and Brazoria (Trende Fig 135, p186), we are offered partisan plots only, with no demographic counterparts.

Taken as a whole, the Trende report does not dispute, and in some cases recapitulates, the strong findings in my report that the state's plans are systematically extreme outliers in their demographic attributes, always in a direction that dilutes voting power for minority groups. I will review the quantitative evidence in §3.2.

### 2.2 The state's enacted plans are highly non-compact

Mr. Trende makes it clear, both explicitly and implicitly, that he judges the state's enacted plans to be very non-compact. Explicitly, as noted before, he argues that Gingles 1 districts should not be judged against the state's plan: "If a state pursues a partisan gerrymander, (and its partisan goals outweigh compactness goals), then its enacted districts may not be very compact" (Trende 115). The fact that the state's plans are not compact is also implicit in his frequent criticisms of the district shapes in alternative plans, even though they are more compact than—and even get their most erratic boundaries directly from—the state's plans.

<sup>5</sup>I fully recognize how difficult it is to organize and communicate information for such a long list of districts under discussion, having faced this challenge myself.

<sup>6</sup>His report says very little about the way the SMC method is deployed. See Appendix C for further discussion.

<sup>7</sup>"However, when we look at the political features of these districts, we see results at least as extreme as the racial features, if not more so." (Trende 42)

### 3 Evidence for racial predominance

#### 3.1 District-by-district shifts

In my report of May 23, I offer numerous instances of district-specific evidence that race predominated over party in the movement of district boundaries. Some examples include the following.

- "The benchmark plan's CD 24 had seen major growth in its coalition population, so that White residents had dropped below half of the voting age population. The new design aggressively reduced the coalition presence—Black-Latino-Asian CVAP was decreased from 40.8% to 25.5%, a difference of 15.3 percentage points. The shift from old to new is much starker on racial than on partisan grounds; the Biden share of the district drops by only 9.1 percentage points." (May 23, p36)
- "The adjustment of [SD] 9 from its pre-redistricting balance shows a much more pronounced racial than partisan differential: coalition CVAP was pared back from 44.4% to 35%, which put it likely out of reach for the minority candidate of choice. At the same time the Biden share fell by only 5.8 percentage points." (May 23, p42)
- "The net change in CVAP with the new [HD 96] lines amounts to –8.3% coalition CVAP, and –5.6% Biden share—a heavier demographic than partisan differential. This brings the district down from 44.4% to 36.1% coalition CVAP. The terrain removed from the district is divided up among districts 90, 95, 97, 101—all ending up with coalition CVAP shares below 30% or over 65%." (May 23, p49)
- "HD 112 hugs the northeast outer boundary of Dallas County, including the most heavily White precincts to secure its overall 33.3% coalition CVAP share—an extremely steep dropoff from the benchmark district, whose coalition population had grown to 51.8% of CVAP. This drop of 18.5 percentage points is not close to matched by the partisan shift, where the Biden support share drops by under 5 points." (May 23, p50)
- "[C]oalition CVAP [in HD 126] was dropped from 55.6% share to 44.9%, a much more pronounced shift than its partisan balance. The territory that was shed in the reconfiguration, itself with a 77% coalition CVAP share, was relocated into neighboring districts 139 and 148, both with roughly 70-80% coalition CVAP share." (May 23, p60)

In these passages, I discuss the net effects of re-drawing on the racial and partisan balance of the districts. By contrast, instead of looking at net shifts in Biden share and POC share of the *districts*, Mr. Trende focuses on something that seems distracting at best: the number of *precincts* with a Biden majority, and Biden's performance in those. (See for instance his Table 1 and Table 2, and many others throughout.) It is hard for me to see how that helps us understand the net effect on the districts, which is of course what matters for electoral performance.

#### 3.2 Extreme outliers

There are seven district clusters that feature both in VRA Section 2 and in constitutional claims in the TX-NAACP complaint. In each one, if we examine the district that has the most coalition CVAP without attaining a majority, we find a striking pattern of "cracking": the demographic makeup is an extreme outlier, having significantly lower minority CVAP than the corresponding district in the ensemble of alternative plans.



- C1 (Congressional Tarrant-Dallas): the state has 39.64% BHA CVAP in CD 6, making that the 4th-indexed district out of 7. Compared to the neutral ensemble, this plan falls in the **4th percentile**. Additionally, CD 12 is 3rd-indexed at 31.69% BHA CVAP—this is in the **2nd percentile** relative to its comparators.
- C2 (Congressional Harris-Ft. Bend): the state has 46.09% BHA CVAP in CD 22, making it 4th-indexed out of 8. This is lower than nearly every one of the 100,000 comparators (**0th percentile**). Note that the ensemble median is 55.79% in this case, and my alternative maps have 56.83% and 52.88% BHA CVAP, respectively. This means that most neutral plans have more majority-minority districts than the enacted plan, and that the alternative plans look more statistically typical.
- S1 (Senate Tarrant-Dallas): the state has 38.10% BHA CVAP in SD 10, making it 6th indexed out of 8. This falls in the **0th percentile**.<sup>8</sup>
- S2 (Senate Ft. Bend): state's SD 18 has 49.55% BHA CVAP, making it 2nd-indexed out of 5. The state is in the **0th percentile**, more extreme than nearly every comparator plan. Note that alternative maps have 53.34 and 61.09% BHA CVAP, respectively, and that the ensemble median is 56.98% BHA CVAP—once again, most of the neutral plans exhibit a district that is comfortably past the majority-minority mark.
- H1 (House Tarrant): state's HD 96 has 36.14% BHA CVAP, making it 7th-indexed out of 11. The state lands in the **0th percentile**, more extreme than every single plan in the ensemble. The median for corresponding districts is over 47% BHA CVAP, meaning that nearly half of the ensemble had an additional majority-minority district. Additionally, the state's 6th-indexed district (HD 99, 33.81% BHA CVAP) also falls in the **0th percentile**.
- H3 (House Denton/Wise): state's highest coalition CVAP is HD 65, with 34.88%. This falls in the **0th percentile**, more extreme than every one of the 100,000 generated plans.
- H5 (House Brazoria): state's HD 25 (46.98% BHA CVAP) is 2nd-indexed, i.e., has higher coalition share than the other district in Brazoria. The state falls in the **1st percentile**. Here, the ensemble median is 53.31% BHA CVAP, and my alternative plan has 56.50%. This means more than half of blindly drawn plans have an additional majority-coalition district, as my plan does.

The state is more extreme than over 96% of neutral alternative plans in every one of these seven instances. In five clusters, the state's demographics are more extreme than over 99.5% of neutral alternatives. (In the bulleted list I have rounded percentile figures to the nearest whole number; Table 1 below contains more detail.)

Next, we examine the hypothesis that the reason for the extreme outlying racial statistics is the pursuit of Republican advantage. Table 1 compares each flagged district above to the full ensemble, and then to the subset that has at least as much partisan advantage. This is assessed in two different ways: from the Trump vs. Biden contest alone, and then from summing over all 19 general elections in my dataset. (For instance, C1 has 7 districts, so there are  $7 \cdot 19 = 133$  contests that have a D or R winner. The state's plan has 77 R wins out of 133. We restrict to ensemble plans that have *at least* 77 R wins.)

<sup>8</sup>I note that the tables from my reports include 7 districts in S2 (SD 9, 10, 12, 16, 22, 23, and 30), while the ensembles include an eighth district (SD 2).

No matter which way Republican advantage is sliced, Table 1 shows that partisanship does not explain the extremely low coalition share in the selected districts. We can see this because the percentiles remain extreme even among the subset that matches or exceeds the state's plan in partisan terms. If partisanship made the low CVAPs reasonably likely, we might see percentiles around 50; if partisanship forced the low CVAPs, we would see percentiles around 100. Instead, they numbers stay near zero.

<i>What share of plans has lower minority CVAP in the corresponding district?</i>							
		full ensemble		at least as Trump-favoring		at least as Republican-favoring	
<b>C1</b>	CD 6	100,000	3.72%	12,427	2.79%	29,139	2.69%
	CD 12		1.55%		0.02%		0.01%
<b>C2</b>	CD 22	100,000	0.05%	30,485	0.10%	74,042	0.06%
<b>S1</b>	SD 10	100,000	0.00%	9539	0.00%	0	—
	SD 2		0.01%		0.06%		—
<b>S2</b>	SD 18	100,000	0.28%	30,497	0.23%	32,269	0.23%
<b>H1</b>	HD 96	100,000	0.00%	98,574	0.00%	16,206	0.00%
	HD 99		0.01%		0.01%		0.00%
<b>H3</b>	HD 65	100,000	0.00%	99,815	0.00%	166	0.00%
<b>H5</b>	HD 25	100,000	0.68%	51,397	1.32%	47,717	1.42%

Table 1: **The state's plan is an extreme outlier in its racial statistics, even controlling for partisanship.** This table explores the possibility that the apparent racial "cracking" in the state's enacted plan can be explained by partisan advantage. Out of the full ensemble of 100,000 generated plans for each cluster, we restrict to only those plans that have *at least as many districts* where Trump beats Biden; then we restrict to only those plans where at least as many districts were won by Republicans across the 19 general election contests overall. For instance, compared to the state's district SD 18 in cluster S2, only 0.28% of the full ensemble has a lower coalition CVAP share. The state's plan has 2 Trump-favoring districts out of 5, and there are 30,497 plans that match or exceed that. If Trump advantage explained the racial skew, then the share with lower CVAP would be unremarkable compared to this batch of plans, perhaps reverting to near 50%. Instead, the low minority CVAP is *even rarer* in this restricted pool—only 0.23% of these Trump-favoring plans have lower minority CVAP. In no case does the mere focus on partisan advantage substantially mitigate the extreme racial statistics in the state's enacted plan.

This look at signs of cracking (systematically examining the state's districts closest to 50% coalition CVAP without hitting that mark) presents extremely strong evidence that race was used by the state as a sorting mechanism. My ensembles, in most instances, find many thousands of alternative plans that achieve at least as strong of a partisan effect; in no case did these plans selected for partisanship tend to show minority CVAP as low as the state. This makes it implausible that the reduced CVAP was a mere incidental byproduct of partisan aims, and more likely that the process was highly race-conscious.

## 4 Notes on EI

In Section VIII (p206-208), Mr. Trende states that he does not offer an analysis of my RPV work, nor does he perform an EI analysis of his own. However, he provides "thoughts" on the topic. These include an opinion that he sources to other authors, holding that EI may not be reliable because, in Mr. Trende's words, "overwhelmingly White, rural Texas precincts provide disproportionate information on the White vote; in sub-county levels, exurban counties are going to provide disproportionate information on the White vote. But experience and the description of shifts here suggest that urban and inner-suburban Whites are quite different in their voting styles." (Trende p208). It is possible that he is quoting authors who were writing about earlier generations of EI, such as King's original *truncated normal* model from the mid-1990s, which fit a global distribution to each group's behavior. The EI carried out in my reports, as I noted, uses the King-Rosen-Tanner multinomial Dirichlet model (introduced in 1999 and refined in the early 2000s), which is *hierarchical*, fitting a different probability distribution to *every precinct*. It is completely false that the presence of very White precincts weakens conclusions about minority preferences; it is also completely false that hierarchical EI methods assume that urban and suburban White voters (say) exhibit the same voting behavior.<sup>9</sup> Furthermore, I present EI outputs at the level of counties and county pairs (e.g., from May 23 report, Table 19 is Tarrant and Dallas; Table 20 is Harris and Ft. Bend, etc). This definitively eliminates any possibility of "exurban counties" influencing the results in any way at all.

These opinions on EI simply have no bearing whatsoever on my work in this case.

## 5 Conclusion

The Trende report reinforces my findings in many places, effectively concurring repeatedly that the state's enacted districting plans are racial outliers and are highly non-compact. The findings of racial outlier status are found using *ensemble methods*, comparing the plans under discussion to large samples of alternatives that were generated in a controlled fashion. The challenged regions of Texas that I discuss in my reports meet even the most extreme standard for demonstrating measurable harm that I am aware of: namely, maps with more minority electoral opportunity would often occur merely by chance. The Texas enacted plans suppress electoral opportunity for Black, Hispanic, and Asian voters, and erase numerous majority-minority districts, *even compared to a baseline given by a substantial share of race-blind trials*. Writing as the state's expert, Mr. Trende presents data that supports this finding, but opines that the dilutive impact may be excused by its confluence with a partisan agenda.

In his discussion of my eleven modular Gingles 1 alternative plans, Mr. Trende only offers the opinion that they are insufficiently compact. He does not dispute that they are generally more compact than the state's enacted plan covering the same terrain (and, incidentally, more compact than the benchmark plan from the last Census cycle as well). He rarely discusses electoral performance in the alternative plans, and when he does it is extremely limited, considering only demographics and the Biden share of the major-party vote in the Presidential contest of 2020. He does not contest my selection of regionally-specific general and primary elections to constitute a score of

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<sup>9</sup>Indeed, my prior published work demonstrates that the hierarchical model is capable of detecting regional differences within a single group's behavior. See for instance Figure 4 and discussion from Becker, Duchin, Gold, and Hirsch, *Computational Redistricting and the Voting Rights Act*, which appeared in Volume 20, Number 4 of the *Election Law Journal* (2021). The figure shows that even a statewide run of the hierarchical EI model in Texas can find that Black voters in the Houston area have quite different preferences from those in the Dallas area.



*electoral alignment* and therefore does not dispute my finding that the alternative maps generally offer meaningfully more opportunity for Black, Hispanic, and Asian voters in Texas to elect candidates of choice.<sup>10</sup>

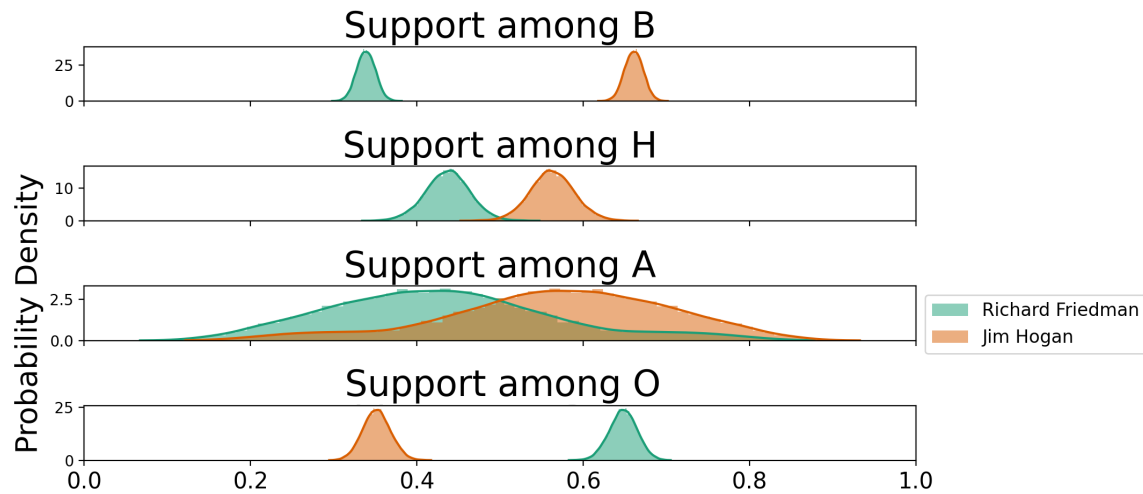
Finally, Mr. Trende's remarks on EI have no connection to the methods in my reports.

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<sup>10</sup>Indeed, on p203, Mr. Trende writes that "Dr. Duchin has not demonstrated that this district [HD 83 in H4-ALT] would elect Democratic candidates." He ignores the supplemental Lubbock plan (H4-ALT-B, June 20, p17) that does just that, emphasizing electoral effectiveness rather than the *Bartlett* demographic threshold of 50%+1 minority CVAP.

## A Confidence in candidate of choice

This appendix presents a second example illustrating the use of confidence scores to identify candidates of choice. (Compare Figure 1.)



Contest	Regional Coalition Candidate of Choice	Conf	Individual Groups			
			B Conf	H Conf	A Conf	O Conf
Ag. Comm. Runoff 2014	Jim Hogan	1	1	0.878	0.624	0

Figure 4: This example is similar to the earlier one, this time reproduced from Figure 2 in my June 20 report. The plots are from EI outputs in Tarrant/Dallas counties, and the table shows corresponding confidence estimates that Hogan is indeed each group's candidate of choice. *This table corrects an error in the earlier report (June 20, p4, Figure 2), where the table values were mistakenly pulled from Harris/Ft. Bend instead of Tarrant/Dallas.*

## B RPV point estimates in counties and county pairs

There are many ways to slice the RPV analysis in a large state like Texas, and my reports have offered quite a few levels of analysis: individual counties, pairs of counties, clusters of districts from the enacted plan, and individual districts in the enacted plan and the alternative plans.

In my June 20 report, Tables 1-10 presented information used to identify candidates of choice using confidence/uncertainty determinations at the level of counties and county pairs. However the point estimates that were presented in Appendix A of that report were conducted on a different level: district clusters from the enacted plan. In each case, the district clusters include most or all of the corresponding counties and county pairs, but also include a large number of rural counties in the area because of the sprawl in the enacted districts. Inclusion of these rural counties changes the point estimates of group preference, as we would expect.

Simply for completeness of the record, I am now reporting the point estimates from the counties and county clusters themselves.

Contest	Candidate of Choice	B Est	H Est	A Est	O4 Est	BHA Est	O2 Est
U.S. Senate Runoff 2012	Paul Sadler	0.504	0.651	0.485	0.946	0.570	0.950
Ag. Comm. Primary 2014	Jim Hogan	0.423	0.426	0.438	0.231	0.425	0.207
Ag. Comm. Runoff 2014	Jim Hogan	0.661	0.563	0.569	0.352	0.631	0.304
Governor Primary 2014	Wendy Davis	0.971	0.683	0.829	0.990	0.905	0.986
RR Comm. Primary 2014	Steve Brown	0.682	0.552	0.553	0.626	0.654	0.608
U.S. Senate Primary 2014	David Alameel	0.479	0.638	0.155	0.689	0.515	0.678
U.S. Senate Runoff 2014	David Alameel	0.708	0.661	0.657	0.960	0.700	0.968
President Primary 2016	Hilary Clinton	0.885	0.608	0.381	0.587	0.801	0.530
RR Comm. Primary 2016	Grady Yarbrough	0.497	0.342	0.357	0.299	0.450	0.275
RR Comm. Runoff 2016	Grady Yarbrough	0.677	0.578	0.468	0.600	0.648	0.586
Comptroller Primary 2018	Tim Mahoney	0.550	0.594	0.409	0.365	0.549	0.357
Governor Runoff 2018	Lupe Valdez	0.736	0.903	0.416	0.547	0.773	0.521
Land Comm. Primary 2018	Miguel Suazo	0.550	0.837	0.580	0.697	0.613	0.697
Lt. Governor Primary 2018	Michael Cooper	0.651	0.640	0.683	0.353	0.652	0.342
RR Comm. Primary 2018	Chris Spellmon	0.620	0.408	0.464	0.417	0.574	0.402
U.S. Senate Primary 2018	Sema Hernandez	0.308	0.531	0.459	0.067	0.378	0.062
RR Comm. Runoff 2020	Roberto Alonzo	0.498	0.640	0.525	0.227	0.514	0.226
U.S. Senate Runoff 2020	Royce West	0.945	0.488	0.466	0.402	0.889	0.351

Table 2: Point estimates for Tarrant-Dallas Democratic primary and runoff elections. (Matches June 20 report, Table 1, p6.)

Contest	Candidate of Choice	B Est	H Est	A Est	O4 Est	BHA Est	O2 Est
U.S. Senate Primary 2012	Grady Yarbrough	0.354	0.327	0.162	0.152	0.341	0.074
U.S. Senate Runoff 2012	Grady Yarbrough	0.597	0.387	0.325	0.050	0.513	0.033
Ag. Comm. Primary 2014	Jim Hogan	0.515	0.511	0.405	0.244	0.511	0.184
Ag. Comm. Runoff 2014	Jim Hogan	0.657	0.598	0.495	0.433	0.636	0.378
Governor Primary 2014	Wendy Davis	0.963	0.660	0.871	0.990	0.896	0.985
RR Comm. Primary 2014	Steve Brown	0.735	0.515	0.657	0.725	0.687	0.703
U.S. Senate Primary 2014	David Alameel	0.517	0.538	0.196	0.796	0.526	0.772
U.S. Senate Runoff 2014	David Alameel	0.625	0.677	0.682	0.973	0.665	0.976
President Primary 2016	Hilary Clinton	0.876	0.664	0.556	0.607	0.800	0.523
RR Comm. Primary 2016	Grady Yarbrough	0.574	0.339	0.270	0.315	0.479	0.237
RR Comm. Runoff 2016	Grady Yarbrough	0.741	0.600	0.494	0.313	0.702	0.176
Comptroller Primary 2018	Tim Mahoney	0.483	0.700	0.562	0.360	0.538	0.357
Governor Runoff 2018	Andrew White	0.799	0.283	0.488	0.808	0.676	0.766
Land Comm. Primary 2018	Miguel Suazo	0.562	0.806	0.731	0.738	0.631	0.754
Lt. Governor Primary 2018	Michael Cooper	0.666	0.657	0.705	0.281	0.670	0.257
RR Comm. Primary 2018	Chris Spellmon	0.569	0.293	0.458	0.305	0.500	0.286
U.S. Senate Primary 2018	Sema Hernandez	0.243	0.536	0.392	0.049	0.341	0.040
President Primary 2020	Joe Biden	0.527	0.155	0.324	0.440	0.418	0.415
RR Comm. Runoff 2020	Roberto Alonzo	0.525	0.645	0.439	0.190	0.540	0.157
U.S. Senate Runoff 2020	Royce West	0.681	0.323	0.470	0.369	0.605	0.343

Table 3: Point estimates for Harris-Ft. Bend Democratic primary and runoff elections. (Matches June 20 report, Table 3, p7.)

Contest	Candidate of Choice	B Est	H Est	A Est	O4 Est	BHA Est	O2 Est
U.S. Senate Primary 2012	Paul Sadler	0.281	0.332	0.430	0.540	0.317	0.509
U.S. Senate Runoff 2012	Paul Sadler	0.657	0.761	0.642	0.884	0.734	0.901
Ag. Comm. Primary 2014	Richard Friedman	0.340	0.444	0.289	0.357	0.404	0.378
Ag. Comm. Runoff 2014	Richard Friedman	0.517	0.587	0.697	0.516	0.616	0.497
Governor Primary 2014	Wendy Davis	0.948	0.849	0.943	0.989	0.868	0.981
RR Comm. Primary 2014	Steve Brown	0.722	0.607	0.710	0.738	0.737	0.674
U.S. Senate Primary 2014	Kesha Rogers	0.450	0.305	0.394	0.153	0.418	0.180
U.S. Senate Runoff 2014	David Alameel	0.775	0.736	0.733	0.858	0.735	0.835
President Primary 2016	Bernie Sanders	0.585	0.611	0.228	0.466	0.501	0.475
RR Comm. Primary 2016	Cody Garrett	0.339	0.390	0.402	0.322	0.384	0.324
RR Comm. Runoff 2016	Grady Yarbrough	0.506	0.526	0.364	0.395	0.516	0.426
Comptroller Primary 2018	Joi Chevalier	0.745	0.495	0.651	0.673	0.653	0.649
Governor Runoff 2018	Lupe Valdez	0.643	0.596	0.542	0.534	0.666	0.548
Land Comm. Primary 2018	Miguel Suazo	0.767	0.571	0.700	0.701	0.706	0.675
Lt. Governor Primary 2018	Michael Cooper	0.491	0.551	0.438	0.305	0.504	0.308
RR Comm. Primary 2018	Roman McAllen	0.654	0.492	0.579	0.575	0.591	0.560
U.S. Senate Primary 2018	Beto O'Rourke	0.584	0.330	0.598	0.875	0.454	0.865
President Primary 2020	Bernie Sanders	0.576	0.553	0.216	0.311	0.445	0.296
RR Comm. Runoff 2020	Chrysta Castaneda	0.609	0.485	0.686	0.828	0.610	0.828
U.S. Senate Primary 2020	Royce West	0.316	0.156	0.317	0.177	0.291	0.166
U.S. Senate Runoff 2020	Royce West	0.699	0.380	0.607	0.373	0.586	0.360

Table 4: Point estimates for Denton-Wise Democratic primary and runoff elections. (Matches June 20 report, Table 5, p8.)

Contest	Candidate of Choice	B Est	H Est	A Est	O4 Est	BHA Est	O2 Est
U.S. Senate Primary 2012	Paul Sadler	0.272	0.384	0.322	0.572	0.333	0.576
U.S. Senate Runoff 2012	Paul Sadler	0.559	0.679	0.631	0.696	0.636	0.756
Ag. Comm. Primary 2014	Jim Hogan	0.404	0.458	0.365	0.402	0.428	0.405
Ag. Comm. Runoff 2014	Jim Hogan	0.491	0.552	0.575	0.325	0.533	0.332
Governor Primary 2014	Wendy Davis	0.806	0.684	0.768	0.960	0.734	0.957
RR Comm. Primary 2014	Steve Brown	0.555	0.586	0.556	0.547	0.570	0.561
U.S. Senate Primary 2014	Maxey Scherr	0.386	0.412	0.378	0.511	0.397	0.496
U.S. Senate Runoff 2014	David Alameel	0.499	0.617	0.481	0.696	0.552	0.695
President Primary 2016	Hillary Clinton	0.651	0.709	0.339	0.502	0.691	0.448
RR Comm. Primary 2016	Cody Garrett	0.418	0.501	0.369	0.480	0.465	0.467
RR Comm. Runoff 2016	Grady Yarbrough	0.613	0.549	0.447	0.578	0.555	0.499
Comptroller Primary 2018	Tim Mahoney	0.536	0.526	0.402	0.427	0.524	0.413
Governor Runoff 2018	Lupe Valdez	0.534	0.640	0.497	0.400	0.584	0.400
Land Comm. Primary 2018	Miguel Suazo	0.606	0.770	0.530	0.617	0.704	0.594
Lt. Governor Primary 2018	Mike Collier	0.467	0.552	0.531	0.723	0.510	0.712
RR Comm. Primary 2018	Roman McAllen	0.547	0.621	0.383	0.444	0.580	0.427
U.S. Senate Primary 2018	Sema Hernandez	0.335	0.443	0.246	0.043	0.407	0.048
President Primary 2020	Bernie Sanders	0.348	0.391	0.332	0.353	0.380	0.348
RR Comm. Runoff 2020	Chrysta Castaneda	0.501	0.565	0.552	0.774	0.530	0.776
U.S. Senate Primary 2020	Annie Garcia	0.288	0.451	0.209	0.231	0.399	0.218
U.S. Senate Runoff 2020	Royce West	0.621	0.403	0.491	0.280	0.513	0.271

Table 5: Point estimates for Lubbock County Democratic primary and runoff elections. (Matches June 20 report, Table 7, p9.)

Contest	Candidate of Choice	B Est	H Est	A Est	O4 Est	BHA Est	O2 Est
U.S. Senate Primary 2012	Paul Sadler	0.307	0.385	0.354	0.516	0.353	0.491
U.S. Senate Runoff 2012	Paul Sadler	0.690	0.759	0.679	0.700	0.733	0.625
Ag. Comm. Primary 2014	Jim Hogan	0.470	0.393	0.420	0.404	0.426	0.381
Ag. Comm. Runoff 2014	Jim Hogan	0.666	0.621	0.634	0.568	0.639	0.562
Governor Primary 2014	Wendy Davis	0.889	0.872	0.932	0.953	0.900	0.932
RR Comm. Primary 2014	Steve Brown	0.717	0.613	0.679	0.713	0.664	0.709
U.S. Senate Primary 2014	David Alameel	0.370	0.557	0.344	0.572	0.448	0.580
U.S. Senate Runoff 2014	David Alameel	0.659	0.813	0.759	0.706	0.759	0.734
President Primary 2016	Hillary Clinton	0.785	0.580	0.698	0.778	0.728	0.665
RR Comm. Primary 2016	Grady Yarbrough	0.475	0.429	0.465	0.469	0.461	0.453
RR Comm. Runoff 2016	Cody Garrett	0.454	0.556	0.533	0.562	0.522	0.481
Comptroller Primary 2018	Joi Chevalier	0.571	0.416	0.656	0.474	0.554	0.498
Governor Runoff 2018	Andrew White	0.669	0.623	0.653	0.683	0.648	0.714
Land Comm. Primary 2018	Miguel Suazo	0.714	0.585	0.728	0.697	0.691	0.681
Lt. Governor Primary 2018	Michael Cooper	0.593	0.428	0.543	0.472	0.525	0.497
RR Comm. Primary 2018	Roman McAllen	0.521	0.552	0.500	0.532	0.522	0.539
U.S. Senate Primary 2018	Beto O'Rourke	0.294	0.442	0.534	0.609	0.428	0.576
President Primary 2020	Joe Biden	0.513	0.264	0.423	0.500	0.421	0.528
RR Comm. Runoff 2020	Chrysta Castaneda	0.490	0.596	0.573	0.657	0.538	0.638
U.S. Senate Primary 2020	Cristina Ramirez	0.176	0.418	0.197	0.254	0.244	0.232
U.S. Senate Runoff 2020	Royce West	0.665	0.290	0.576	0.447	0.565	0.487

Table 6: Point estimates for Brazoria County Democratic primary and runoff elections. (Matches June 20 report, Table 9, p10.)

## C Issues with Trende ensemble methodology

As discussed above, the ensembles in the Trende report are made with software implementing a Sequential Monte Carlo (SMC) methodology, which has not yet completed peer review. I do not believe that Mr. Trende is claiming expertise in the underlying theory or in assessing sampling diagnostics. Because the report describes concordance with my own ensemble results by way of validation, I did not undertake a detailed review of the methods or resulting data. However, this should not be interpreted as an endorsement of the ensemble data practices, and I will briefly outline a few potential issues.

As an initial and significant issue, the Trende report does not describe which districting principles are operationalized in his SMC runs. For instance, the only mention I can find of a priority for keeping counties whole is a passing reference on page 188 to El Paso-area state House districts. We do not know what population deviation tolerance was enforced, whether a compactness target was set, or what tests of robustness and sample quality were conducted. Without these elements, we can not secure confidence in the resulting analysis.

In addition, his "dotplots" do not show the median, quartiles, or any other marker of statistical position in the columns. They do, however, show visual features that are flags of sample redundancy. Previous experts who have examined Trende's ensembles have indeed found this to be the case; in the New York state Senate litigation, a replication effort found that *more than half* of the districts produced by Trende's study design appeared exactly identically in *more than half* of the plans.<sup>11</sup> If SMC is run to specification, the draws should be approximately uncorrelated, and this should not occur.

The figures in the Trende report do not enable us to diagnose the sample redundancy problem quantitatively, but they do show visual artifacts of redundancy. Here, in Figure 5, I highlight evidence that certain values appear hundreds or even thousands of times in the Trende ensembles, as we would expect to see from a poorly calibrated SMC ensemble.

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<sup>11</sup>*Harkenrider v. Hochul*, Index No. E2022-0116CV, Second Affidavit of Kristopher Tapp.

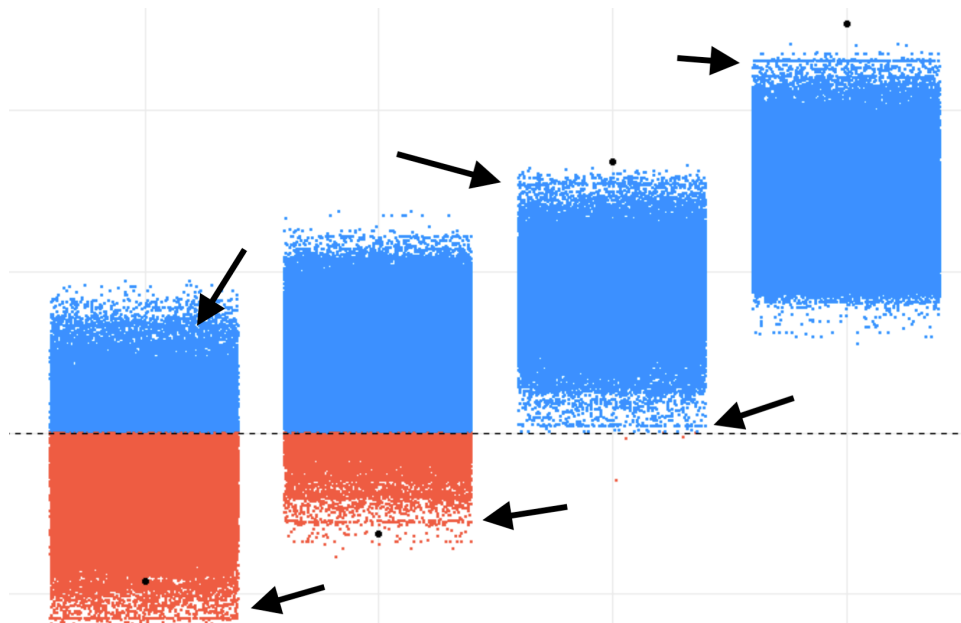



Figure 5: The dotplots exhibited in the Trende report do not make it possible to discern where the median value lies—it could reasonably be anywhere in the bulk of the colored dots. Furthermore, the presence of horizontal lines in the plot (which I have marked with arrows) show the high incidence of repeated districts: a single district may appear hundreds or even tens of thousands of times if the ensemble has not been properly checked for sample diversity. Clear indicators of sample redundancy (identical districts appearing thousands of times) are visible in every column of this excerpt from his Figure 27 (Trende p43).

I reserve the right to continue to supplement my report in light of additional facts, testimony and/or materials that may come to light. Pursuant to 28 U.S.C. 1746, I declare under penalty of perjury of the laws of the United States that the foregoing is true and correct according to the best of my knowledge, information, and belief.

Executed this 1st day of August, 2022.

  
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Dr. Moon Duchin